Properties of Logarithmic Functions

Product Rule for Logarithms (pg. 610 – 615)
For any positive numbers M, N, and *a* (*a* ≠ 1), $\log_a(MN) = \log_a M + \log_a N$
(The logarithm of a product is the sum of the logarithms of the factors.

Examples:
Express as an equivalent expression that is a sum of logarithms.
$\log_2(4\cdot 16)$

Express as an equivalent expression that is a single logarithm.
$\log_b 7 + \log_b 5$

Logarithm of Powers:
For any positive numbers M, N, and *a* (*a* ≠ 1) and any real number P, $\log_a M^P = P\cdot\log_a M$
(The logarithm of a power of M is the exponent times the logarithm of M.

Examples:
Use the power rule for logarithms to write an equivalent expression that is a product.

$\log_a 9^{-5}$ $\log_7 \sqrt[3]{x}$

Logarithms of Quotients
For any positive numbers M, N, and *a* (*a* ≠ 1), $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$
(The logarithm of a quotient is the logarithm of the dividend minus the logarithm of the divisor.

Examples:
Express as an equivalent expression that is a difference of logarithms.
$\log_t\left(\frac{6}{U}\right)$

Express as an equivalent expression that is a single logarithm.
$\log_b 17 - \log_b 27$

Using the Properties Together
Examples: Express as an equivalent expression, using the individual logarithms of *x*, *y*, and *z*.

$\log_b \frac{x^3}{yz}$ $\log_a \sqrt[4]{\frac{xy}{z^3}}$

Express as an equivalent expression that is a single logarithm.

$\frac{1}{2}\log_a x - 7\log_a y + \log_a z$

$\log_a\left(\frac{b}{\sqrt{x}}\right) + \log_a \sqrt{bx}$

Given $\log_a 2 = .431$ *and* $\log_a 3 = .683$ Use the properties of logarithms to calculate a value for each of the following. If this is not possible, state so.

$\log_a 6$

$\log_a \frac{2}{3}$

$\log_a 81$

$\log_a \frac{1}{3}$

$\log_a (2a)$

$\log_a 5$

The logarithm of the Base to an Exponent

For any base *a*, $\log_a a^k = k$.

(The logarithm, base *a*, of *a* to an exponent is the exponent.)

Examples:

$\log_3 3^7$

$\log_{10} 10^{-5.2}$